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Grassmannian Duality and the Particle Spectrum

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Schemes based on anticommuting scalar coordinates, corresponding to properties, lead to generations of particles naturally. The application of Grassmannian duality cuts down the number of states substantially and is vital for constructing sensible Lagrangians anyhow. We apply duality to all of the subgroups within the *classification* group $SU(3) \times SU(2)_L \times SU(2)_R$, which encompasses the standard model gauge group, and thereby determine the full state inventory; this includes the definite prediction of quarks with charge $-4/3$ and other exotic states. Assuming universal gravitational coupling to the gauge fields and parity even property curvature, we also obtain $4 \sin^2 \theta_w = 1 - 2\alpha/3\alpha_s$ which is not far from the experimental value around the M_Z mass.

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1. Generations and Duality

The existence of generations of particle families is one of the deepest conundrums in theoretical physics. Who can forget Rabi's exclamation upon the discovery of the muon: "Who ordered that?". The invocation of a new family symmetry group offers no explanation to Rabi but can serve as a useful classification tool and it can even be gauged. Over the last few years we have suggested that the full description of events through a spacetime x enlarged by anticommuting complex scalar coordinates ζ , which correspond to fundamental properties, can serve to explain naturally the repetition of particle families. Put another way, we have proposed that the attributes of events represent the extra dimensions which everyone is chasing; such an approach is distinct from other attempts at unification. We have gone on to show that the forces and their parity violation (at low energies anyway) can be accommodated through the curvature of the metric in the enlarged spacetime $X = (x, \zeta, \bar{\zeta})$ particularly via the $x - \zeta$ sector, making a distinction between left and right attributes.

The article¹ on "The Force and Gravity of Events" contains a detailed reference list describing progressive steps made in researching this scenario. Our investigations have shown that a bare minimum of three (parity blind) colour properties $\zeta^1, \zeta^2, \zeta^3$ plus two electroweak properties ζ^0, ζ^4 , *further distinguished by their parities*, can potentially produce the known generations and can even describe the standard model (SM) interactions correctly; the force fields arise through the gauge mesons

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embedded in the frame vectors that connect spacetime to property at the same time that gravitational properties in the spacetime sector stay impartial with respect to handedness, at least semiclassically. On several occasions¹ we have alluded to some of the multiplets engendered by such a scheme, but we have never provided a *full* inventory of the states which ensue. That is the purpose of this article as well as constructing the interactions between all the fields posited by the standard model.

The state zoo predicted by such a scheme arises when the superfields $\Phi(X)$ & $\Psi(X)$ are expanded in ζ and its conjugate $\bar{\zeta}$. There one encounters a *finite* number of terms $(\bar{\zeta})^r(\zeta)^s\phi_{(r,s)}$ & $(\bar{\zeta})^r(\zeta)^s\psi_{(r,s)}$, where r, s run up to the number of properties, with due care taken to obey the spin-statistics theorem. Even so, a considerable number of expansion terms are produced even with only $N = 5$ properties. However once we realise that unitary representations ϕ, ψ are unaffected by the ‘duality’ substitution $(r, s) \rightarrow (N - s, N - r)$, we can apply a duality symmetry to reduce substantially the number of terms in the superfield expansions. Interestingly, the imposition of duality is essential to obtain sensible Lagrangians for the component fields after integrating over property. This is explained at length in the next section as well as the appendix. We make particular choices of duality for each of the subgroups of the SM and, following that, enumerate the full list of right and left states in section 3, paying particular heed to the many chargeless Higgs fields held in Φ since they are responsible for mass generation of bosons and fermions through the Yukawa interaction. The use of a universal property curvature polynomial leads to a determination of the weak mixing angle by demanding universal gravitational coupling to the gauge fields; the value of that angle is close to experimental measurements before taking account of quantum corrections. We conclude with some remarks about this whole approach in the final section.

2. The Need for Duality: Examples

2.1. *One property*

We start with just a single property, corresponding to one complex ζ , representing electricity. Paying no regard to parity at this stage and reminding ourselves that bosons are associated with even powers and fermions with odd powers, the expansions of the superfields in ζ become rather trivial:

$$\begin{aligned}\Phi(x, \zeta, \bar{\zeta}) &= A(x) + \bar{\zeta}\zeta B(x) \\ \Psi(x, \zeta, \bar{\zeta}) &= \bar{\zeta}\psi(x) + \psi^c(x)\zeta \\ \bar{\Psi}(x, \zeta, \bar{\zeta}) &= -\bar{\psi}\zeta + \bar{\zeta}\bar{\psi}^c.\end{aligned}\tag{1}$$

Note that Φ and Ψ (the latter carries a spinorial index) are both bosonic but their expansion coefficients have the correct spin-statistics connection. We shall be integrating over ζ so we will use the convention^a that $\int(d\zeta d\bar{\zeta})(\bar{\zeta}\zeta) = 1$. It follows

^a When dealing with N properties, consult the Appendix for our convention.

that the integrated bilinear

$$\int (d\zeta d\bar{\zeta}) \Phi \Phi = 2AB,$$

can hardly serve as a suitable Lagrangian. This is where the concept of Grassmannian duality comes to the rescue. With only one property, dualization, indicated by the superscript \times , corresponds to

$$1^\times = \bar{\zeta}\zeta, \quad (\bar{\zeta}\zeta)^\times = 1, \quad \zeta^\times = \zeta, \quad (\bar{\zeta})^\times = \bar{\zeta}. \quad (2)$$

Therefore the selfdual ζ polynomials are $(1 + \bar{\zeta}\zeta)$ and ζ , while the anti-selfdual polynomial is $(1 - \bar{\zeta}\zeta)$ with $\zeta \rightarrow 0$. Requiring selfduality means setting $A = B$ and permits suitable Lagrangian bilinears:

$$\int (d\zeta d\bar{\zeta}) \Phi \Phi = 2A^2, \quad \int (d\zeta d\bar{\zeta}) \bar{\Psi} \Psi = \bar{\psi} \psi + \overline{\psi^c} \psi^c = 2\bar{\psi} \psi.$$

Double dualization amounts to the identity operation, and self-evidently the total number of component fields for anti-selfdual plus selfdual adds up to what one would get without imposing those conditions.

2.2. Two properties

Our starting point is the pair of properties, ζ^0 signifying neutrinicity and ζ^4 signifying charge -1 leptonicity; for the present we disregard colour and therefore strong interactions (to which we presently assign the labels $\zeta^{1,2,3}$, signifying colour down-type quarks). Later on we have to distinguish between left and right leptonicity.

Here we are dealing with an $SU(2)$ subgroup of $Sp(4)$. When we expand the superfields in powers $(\bar{\zeta})^s(\zeta)^r$ with r, s running from 0 to 2 we obtain the square given in Table 1 where it will be seen that reflection about the diagonal corresponds to charge conjugation, whereas reflection about the cross-diagonal corresponds to taking the dual.

Table 1. Isospin states $(\bar{\zeta})^r(\zeta)^s$; $r + s$ odd for fermions, $r + s$ even for bosons. Numerical entries indicate dimensions of representations.

$s \backslash r$	0	1	2
0	1	2	1
1	2	$1 \oplus 3$	2
2	1	2	1

With regard to duality signs, we use the antisymmetric Levi-Civita symbols, $\epsilon^{\bar{0}4} = 1, \epsilon^{40} = 1$ with $\epsilon^{\bar{\lambda}\bar{\mu}} \epsilon^{\mu\nu} = \delta^{\bar{\lambda}\nu}$. Introduce the abbreviation $Z \equiv \zeta^{\bar{\mu}} \zeta^{\mu} \equiv \bar{\zeta}\zeta$, so

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$Z^2/2 = \zeta^{\bar{4}}\zeta^4\zeta^{\bar{0}}\zeta^0$. Then the duality operation results in:

$$\begin{aligned} 1^\times &= Z^2/2, & (\zeta^\mu)^\times &= Z\zeta^\mu, & (\zeta^0\zeta^4)^\times &= -(\zeta^0\zeta^4)^\times \\ (\zeta^{\bar{0}}\zeta^0)^\times &= -\zeta^{\bar{4}}\zeta^4, & \text{so } Z^\times &= -Z, & \text{while for the triplet,} \\ (\zeta^{\bar{0}}\zeta^4)^\times &= \zeta^{\bar{0}}\zeta^4, & (\zeta^{\bar{0}}\zeta^0 - \zeta^{\bar{4}}\zeta^4)^\times &= (\zeta^{\bar{0}}\zeta^0 - \zeta^{\bar{4}}\zeta^4), & (\zeta^{\bar{4}}\zeta^0)^\times &= \zeta^{\bar{4}}\zeta^0, \end{aligned} \quad (3)$$

plus all their adjoints. This then allows us to separate the two possibilities^b:

2.2.1. *Selfdual expansion*

For fermions we get

$$\begin{aligned} \sqrt{2}\Psi &= \zeta^{\bar{0}}(1+Z)\nu + \zeta^{\bar{4}}(1+Z)\ell, \\ -\sqrt{2}\bar{\Psi} &\equiv \bar{\nu}(1+Z)\zeta^0 + \bar{\ell}(1+Z)\zeta^4, \end{aligned} \quad (4)$$

ignoring conjugates which only serve to double the final results. The selfdual Bose superfield expansion on the other hand reads

$$\sqrt{2}\Phi = \phi(1+Z^2/2) + \zeta^{\bar{\mu}}\rho^{\mu\nu}\zeta^\nu \quad (5)$$

where ϕ is a singlet and ρ is an isotriplet. These fields comprise eight components in toto.

2.2.2. *Anti-selfdual expansion*

Here we get instead

$$\begin{aligned} \sqrt{2}\Psi &= \zeta^{\bar{0}}(1-Z)\nu + \zeta^{\bar{4}}(1-Z)\ell, \\ \sqrt{2}\bar{\Psi} &\equiv \bar{\nu}(1-Z)\zeta^0 + \bar{\ell}(1-Z)\zeta^4, \end{aligned} \quad (6)$$

plus conjugates, and

$$\Phi = \phi(1-Z^2/2) + \sigma Z + \zeta^{\bar{4}}\zeta^{\bar{0}}\varphi + \bar{\varphi}\zeta^0\zeta^4, \quad (7)$$

where φ is a complex isosinglet. This set also contains eight components. Added to the previous (selfdual) set we get a total of 16 states, agreeing with the number of coefficients expected for these expansions without imposing duality.

2.2.3. *Right and Left*

Presently we will be distinguishing between left and right properties ζ_L, ζ_R as demanded by the gauge interactions of the standard model; so the above analysis can be applied to each of the $SU(2)$ *classification* subgroups in the overarching group $SU(2)_R \times SU(2)_L$, though specifically right-handed gauge fields do not exist in the standard model.

^b In a previous paper we differed from the convention here in that our integration measure had the opposite sign. Thus what we regarded as anti-selfdual then becomes selfdual now.

Thus the allowed *selfdual* combinations with their dimensionalities, for each chirality, are

$$\begin{aligned} \text{Bosons } B^D : - & \quad \mathbf{1} : (1 + Z^2/2), \quad \mathbf{3} : \bar{\zeta}\zeta \\ \& \quad \text{Fermions } F^D : - & \quad \bar{\mathbf{2}} : \bar{\zeta}(1 + Z), \quad \mathbf{2} : \zeta(1 + Z), \end{aligned} \quad (8)$$

with the singlet $\zeta^0\zeta^4$ eliminated. With reference to the triplet, it has the three components $(\zeta^4\zeta^0, [\zeta^4\zeta^4 - \zeta^0\zeta^0]/\sqrt{2}, -\zeta^0\zeta^4)$ carrying charges $(1, 0, -1)$. It is to be understood that this shorthand for the triplet, viz. $\bar{\zeta}\zeta$, will appear subsequently for each of the left or right SU(2) subgroups.

On the other hand, the *anti-selfdual* combinations with their dimensionalities, for each chirality, are

$$\begin{aligned} \text{Bosons } B^A : - & \quad \mathbf{1} : (1 - Z^2/2), \quad Z, \quad \zeta^0\zeta^4, \quad \zeta^4\zeta^0 \\ \& \quad \text{Fermions } F^A : - & \quad \bar{\mathbf{2}} : \bar{\zeta}(1 - Z), \quad \mathbf{2} : \zeta(1 - Z). \end{aligned} \quad (9)$$

2.3. Three properties

We now turn to the strong interaction sector and colour to see what selfduality has to say about this. Table 2 is the analogue of Table 1, before imposition of duality, and contains the following entries for the SU(3) representations arising from $\zeta, \bar{\zeta}$ expansion.

Table 2. The colour multiplets $(\bar{\zeta})^s(\zeta)^r$; $r + s$ odd for fermions, even for bosons. Numerical entries indicate dimensions of representations.

$s \backslash r$	0	1	2	3
0	1	3	$\bar{3}$	1
1	3	$1 \oplus 8$	$3 \oplus \bar{6}$	3
2	3	$\bar{3} \oplus 6$	$1 \oplus 8$	3
3	1	3	$\bar{3}$	1

Observe that the $\bar{6}$ -fold colour multiplet contains such components as $\zeta^1\zeta^2\zeta^3, \zeta^1(\zeta^2\zeta^2 - \zeta^3\zeta^3)$, in contrast to the triplet $\zeta^1(\zeta^2\zeta^2 + \zeta^3\zeta^3) = \zeta^1 Z$, where $Z \equiv (\zeta^1\zeta^1 + \zeta^2\zeta^2 + \zeta^3\zeta^3)$. The duality signs are encapsulated by the typical entries:

$$\begin{aligned} 1^\times &= Z^3/6, \quad Z^\times = Z^2/2, \quad \zeta^2\zeta^1 = -Z\zeta^2\zeta^1 \text{ (octet)} \\ (\zeta^1\zeta^2)^\times &= Z(\zeta^1\zeta^2), \quad (\zeta^1\zeta^2\zeta^3)^\times = \zeta^1\zeta^2\zeta^3, \quad (\zeta^1 Z)^\times = \zeta^1 Z \\ (\zeta^1\zeta^2\zeta^3)^\times &= \zeta^1\zeta^2\zeta^3, \quad (\zeta^1(\zeta^2\zeta^2 - \zeta^3\zeta^3))^\times = \zeta^1(\zeta^2\zeta^2 - \zeta^3\zeta^3) \text{ (sextet)}. \end{aligned} \quad (10)$$

The consequence is that the *selfdual* combinations for 3 properties (including con-

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jugates), with their SU(3) dimensionalities comprise the sets:

$$\begin{aligned} \text{Bosons } B^D : - & \quad \mathbf{1} : (1 + Z^3/6), \quad Z(1 + Z/2), \\ & \quad \bar{\mathbf{3}} : \zeta\zeta(1 + Z), \quad \mathbf{3} : \bar{\zeta}\bar{\zeta}(1 + Z), \quad \mathbf{8} : \zeta\bar{\zeta}(1 - Z); \end{aligned} \quad (11)$$

$$\begin{aligned} \& \quad \text{Fermions } F^D : - & \quad \mathbf{3} : \zeta(1 + Z^2/2), \quad \bar{\mathbf{3}} : \bar{\zeta}(1 + Z^2/2), \\ & \quad \bar{\mathbf{6}} : \zeta\zeta\bar{\zeta}, \quad \mathbf{6} : \zeta\bar{\zeta}\bar{\zeta}, \quad \mathbf{1} : \zeta\zeta\zeta, \quad \bar{\zeta}\bar{\zeta}\bar{\zeta}, \end{aligned} \quad (12)$$

with the triplet ζZ eliminated. This makes for a total of 36 selfdual states. The *anti-selfdual states* number 28 states:

$$\begin{aligned} \text{Bosons } B^A : - & \quad \mathbf{1} : (1 - Z^3/6), \quad Z(1 - Z/2), \\ & \quad \bar{\mathbf{3}} : \zeta\zeta(1 - Z), \quad \mathbf{3} : \bar{\zeta}\bar{\zeta}(1 - Z), \quad \mathbf{8} : \zeta\bar{\zeta}(1 + Z); \end{aligned} \quad (13)$$

$$\begin{aligned} \& \quad \text{Fermions } F^A : - & \quad \mathbf{3} : \zeta(1 - Z^2/2), \quad \zeta Z \\ & \quad \bar{\mathbf{3}} : \bar{\zeta}(1 - Z^2/2), \quad \bar{\zeta} Z, \end{aligned} \quad (14)$$

with the singlet $\zeta\zeta\zeta$ excised. Added to the selfdual ones, we get a total of 64 states, as expected. (For N properties there are 4^N states before constraining them by duality.)

Subsequently, when constructing Lagrangians, it is important to point out that the integrals over property of quadratic combinations such as $B^A B^D$ & $\bar{F}^D F^A$ vanish identically. However cubic terms, which arise when considering Yukawa couplings, such as $B^A B^A B^D$ & $B^A \bar{F}^A F^D$, survive property integration. That applies to each of the subgroups, being independent of the number of properties.

3. The full particle inventory

At this point we can combine all of the attributes and determine the complete set of states, taking into account parity too – we did not do this previously when we just considered the grand unified group SU(5) duality constraints. In order to decide what type of duality is to be applied to each of the standard model subgroups, we must make sure that certain ‘unwanted’ combinations are duly expunged and that we cover the known generations of quarks and leptons, as well as incorporating proper Higgs-like states. Another important consideration is that the fermionic states have to be constrained to be of two types, purely left or purely right; we cannot entertain a schizophrenic or ambidextrous possibility of mixing these two sorts of chirality *unless one of them is an ineffective singlet* in the product. On the contrary, the scalar fields *must bridge right and left*, which may dictate a different choice of duality from the fermions.

So let us start off with the quarks: colour triplets and weak isodoublets. Let ζ_c refer to chromic property (which strictly speaking should carry contravariant labels 1,2,3). When we paid no heed to chirality and ignored duality we were able to identify two typical (U, D) cases: $(\bar{\zeta}_c \bar{\zeta}_c \zeta^0, \bar{\zeta}_c \bar{\zeta}_c \zeta^4)$ and $(\zeta_c \zeta^4 \zeta^0, \zeta_c \zeta^4 \zeta^4)$ or $\zeta_c \zeta^0 \zeta^0$ as having the correct colour and charge quantum numbers. Chromic duality will double these pairs (whether we pick colour dual or antidual) and with the first

choice we can readily impose left or right-handed characteristics on the leptonic component. The second choice is more problematic because it involves a lepton-antilepton product. These must both be left *or* right according to our criterion, so referring to Sec.2.2.3 and (8) we have to take the *triplet* combination; this dictates selfduality for left. The right products lepton-antilepton accompanying them must not jeopardize the chiral purity and must therefore remain right singlets so, as in (9), we have to pick anti-selfduality for right. When we reverse overall chirality, similar sorts of selections must prevail.

Turning to the scalar bosons, we note that $\zeta^1\zeta^2\zeta^3\zeta^4$ has the potential to serve as a Higgs boson. However it is apparent that a similar combination, $\zeta^1\zeta^2\zeta^3\zeta^4$, is a state with $F = 2, Q = -2$ and is undesirable. Besides, such a state would be either left or right and could not bridge chirality. Noting that the colour singlet $\zeta^1\zeta^2\zeta^3$ disappears when we adopt colour anti-selfduality this then indicates chromic anti-selfduality as being the proper fermionic choice. We shall be guided by these considerations below.

Let $F_c^A, B_c^A, F_L^A, B_L^A, F_R^A, B_R^A$ signify the fermionic and bosonic parts of the combinations complying with antiduality, while F_c^D, B_c^D etc. signify those combinations which are selfdual. Then the full fundamental fermionic elements consist of the products $F_c^A F_R^A F_L^D, F_c^A B_R^A B_L^D, B_c^A F_R^A B_L^D, B_c^A B_R^A F_L^D$, in addition to the corresponding terms where one interchanges L and R . The resulting buildup is based upon stating the attributes of the fundamental properties determined by the action of the standard gauge fields, held in the spacetime-property sector of the metric. These property attributes are listed in Table 3. The state inventory comes about through

Table 3. The quantum numbers for weak left isospin, hypercharge and charge of the basic properties. The conjugates $\bar{\zeta}$ naturally have the opposite quantum numbers.

Property	T_{3L}	Y	Q
ζ_L^0	1/2	-1	0
ζ_L^4	-1/2	-1	-1
ζ_R^0	0	0	0
ζ_R^4	0	-2	-1
$\zeta^{1,2,3}$	0	-2/3	-1/3

combinations of these properties. It is noteworthy that sums of these basic attributes correspond exactly to the quantum number assignments made in the SM. Before compiling all these products systematically, it pays to identify where the usual up-, down- quarks and leptons reside, ignoring duality and handedness for now and focussing on their charges. We have already identified two triplet colour combinations that serve that purpose, namely $(\bar{\zeta}_c\bar{\zeta}_c\zeta^0, \bar{\zeta}_c\bar{\zeta}_c\zeta^4)$ and $(\zeta_c\zeta^4\zeta^0, \zeta_c\zeta^4\zeta^4/\zeta_c\zeta^0\zeta^0)$. This highlights the point that *we should expect a new colour triplet quark of charge -4/3*, identified as the combination $\zeta_c\bar{\zeta}^0\zeta^4$, which is part of a weak isotriplet. Turning to

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the leptons, $\mathcal{L} \equiv (\nu, \ell)$ are of course based on dualized generations of fundamental properties (ζ^0, ζ^4) . where it should be noted from (8) that $\zeta_L^0 \zeta_L^4$ and $\zeta_R^0 \zeta_R^4$ vanish for a selfdual choice.

3.1. The Fermionic States

As explained above, possible combinations to be considered are multiplicative sets $F_c^A B_R^D B_L^A, F_c^A B_R^A B_L^D, B_c^A B_R^D F_L^A, B_c^A F_R^A B_L^D$. These are best summarised by listing the direct product representations and the dimensions of the 3 *classification* groups: $SU(3)_c \otimes SU(2)_R \otimes SU(2)_L$; their associated properties are stated in eqs. (8),(9),(11) and (12). In this way we obtain the left fermion combinations (including conjugates)

$$F_c^A B_R^A B_L^D \supset \psi_L \sim \{3, 3, \bar{3}, \bar{3}\} \otimes \{1, 1\} \otimes \{1, 3\} \quad (15)$$

$$B_c^A B_R^A F_L^D \supset \{1, 1, \bar{3}, 3, 8\} \otimes \{1, 1\} \otimes \{\bar{2}, 2\} \quad (16)$$

while the right combinations are

$$F_c^A B_R^D B_L^A \supset \psi_R \sim \{3, 3, \bar{3}, \bar{3}\} \otimes \{1, 3\} \otimes \{1, 1\} \quad (17)$$

$$B_c^A F_R^D B_L^A \supset \{1, 1, \bar{3}, 3, 8\} \otimes \{2, \bar{2}\} \otimes \{1, 1\}. \quad (18)$$

Identifying the quarks and paying little heed to the sizes of the $SU(2)_R$ representations since they are not gauged we can pick out the left quark products:

$$\{3, 3\} \otimes \{1, 1\} \otimes 3 : \zeta \{1 - Z^2/2, Z\} \cdot \{(1 - Z_R^2/2), Z_R\} \cdot \zeta_L \bar{\zeta}_L$$

$$3 \otimes \{1, 1\} \otimes 2 : \bar{\zeta} \bar{\zeta} (1 - Z) \cdot \{(1 - Z_R^2/2), Z_R\} \cdot \zeta_L (1 + Z_L).$$

with similar products where left and right are interchanged (plus their conjugates). The first and second generation of quarks correspond to weak isodoublets but the other quark generations occur in weak isotriplets! So far as leptons are concerned we have to seek colour singlets so must study the set

$$B_c^A B_R^A F_L^D \supset \psi_L \sim \{1, 1\} \otimes \{1, 1\} \otimes \{\bar{2}, 2\} \quad (19)$$

$$B_c^A F_R^D B_L^A \supset \psi_R \sim \{1, 1\} \otimes \{\bar{2}, 2\} \otimes \{1, 1\}, \quad (20)$$

corresponding to the leptonic products

$$\psi_L \sim \{(1 - Z^3/6), Z(1 - Z/2)\} \cdot \{(1 - Z_R^2/2), Z_R\} \cdot \zeta_L (1 - Z_L)$$

$$\psi_R \sim \{(1 - Z^3/6), Z(1 - Z/2)\} \cdot \zeta_R (1 - Z_R) \cdot \{(1 - Z_L^2/2), Z_L\}.$$

We recognize *four* generations of leptons; so here is another prediction of this scheme.

3.2. The Bosonic States

Here one is obliged to take the following dual multiplicative combinations so as to bridge left and right. Relying upon anti-selfdual combinations for the chromic sector, and because $B_R^D B_L^A, B_R^A B_L^D$ does not lead to ambidextrous products, we might be tempted to consider

$$\begin{aligned} B_c^A F_R^A F_L^D : \phi &\sim \{1, 1, \bar{3}, 3, 8\} \otimes \{\bar{2}, 2\} \otimes \{\bar{2}, 2\} \\ B_c^A F_R^D F_L^A : &\quad \{1, 1, \bar{3}, 3, 8\} \otimes \{\bar{2}, 2\} \otimes \{\bar{2}, 2\}, \end{aligned}$$

with weak quartets contained within the colour singlet set:

$$\begin{aligned} B_c^A F_R^A F_L^D &\supset \{(1 - Z^3/6), Z(1 - Z/2)\} \cdot \{\zeta_R, \bar{\zeta}_R\}(1 - Z_R) \cdot \{\zeta_L, \bar{\zeta}_L\}(1 + Z_L) \\ B_c^A F_R^D F_L^A &\supset \{(1 - Z^3/6), Z(1 - Z/2)\} \cdot \{\zeta_R, \bar{\zeta}_R\}(1 + Z_R) \cdot \{\zeta_L, \bar{\zeta}_L\}(1 - Z_L). \end{aligned}$$

However it turns out that the above choices do *not* enjoy Yukawa interactions with the quark and lepton generations (at least in flat space) because of property integration rules. Instead for bosons we are led to relaxing the condition that the chromic sector must be anti-selfdual and consider the following bosonic combinations which are *overall* selfdual and which *do* interact with the fermions:

$$B_c^D F_R^D F_L^D : \phi \sim \{1, 1, \bar{3}, 3, 8\} \otimes \{\bar{2}, 2\} \otimes \{\bar{2}, 2\} \quad (21)$$

$$B_c^D F_R^A F_L^A : \quad \{1, 1, \bar{3}, 3, 8\} \otimes \{\bar{2}, 2\} \otimes \{\bar{2}, 2\}. \quad (22)$$

Here the Higgs fields are contained within the colour singlet set,

$$B_c^A F_R^A F_L^D \supset \{(1 + Z^3/6), Z(1 + Z/2)\} \cdot \{\zeta_R, \bar{\zeta}_R\}(1 + Z_R) \cdot \{\zeta_L, \bar{\zeta}_L\}(1 + Z_L) \quad (23)$$

$$B_c^A F_R^D F_L^A \supset \{(1 + Z^3/6), Z(1 + Z/2)\} \cdot \{\zeta_R, \bar{\zeta}_R\}(1 - Z_R) \cdot \{\zeta_L, \bar{\zeta}_L\}(1 - Z_L). \quad (24)$$

This last selection will become clearer when we consider gauge field and Yukawa interactions where we also arrange that the superfield Φ is even under left-right interchange.

4. Interactions

As there appear to be a menagerie of exotic states (for which there is as yet no evidence) as well as quasi-conventional states, let us just focus on the standard particles that are contained in amongst the tables of bosons and fermions listed in the previous section. We should remind ourselves that the only gauged groups are QCD and the electroweak group, though other researchers — keen on left right symmetry — might advocate more gauging. We need to check that this subset of states^c enjoy interactions expected of them through the metric element connecting property to spacetime. But before launching into that we also need to ascertain that the gauge field Lagrangians themselves emerge correctly from the generalized scalar curvature of the extended space.

^cWe will include the quark family comprising charge $-4/3$, as that is one of the prominent predictions of the present scheme.

4.1. Gauge field interactions

We therefore turn to the extended metric which permits the following general form respecting gauge invariance. Referring to Table 3,

$$x - x \text{ sector, } G_{mn} = g_{mn}C + "l^2\bar{\zeta}(A_m A_n + A_n A_m)\zeta C'/2", \quad (25)$$

$$x - \zeta_L \text{ sector, } G_{m\zeta_L} = -il^2\bar{\zeta}_L(gW_m \cdot \tau - g'B_m)C'/4, \quad (26)$$

$$x - \zeta_R \text{ sector, } G_{m\zeta_R} = -il^2\bar{\zeta}_R g'B_m(\tau_3 - 1)/C'/4 \quad (27)$$

$$x - \zeta \text{ sector, } G_{m\zeta} = -il^2\bar{\zeta}(fV_m \cdot \lambda - \frac{2}{3}g'B_m)C'/4 \quad (28)$$

$$\zeta - \bar{\zeta} \text{ sector, } G_{\mu\bar{\nu}} = l^2\delta_\mu{}^\nu C'/2. \quad (29)$$

Above, W refers to the left weak boson field (with coupling constant g), B to its hypercharge counterpart (with coupling constant $g'/2$), V to the gluon field (with coupling constant f), λ to the SU(3) lambda matrices; A is a generic full vector set comprising all those components multiplied by their coupling constants. C and C' are in principle independent curvature polynomials involving the various invariants $\bar{\zeta}\zeta$ associated with the gauge subgroups. To simplify the argument we will take $C = C'$ to be universal for gauge fields and gravity as this has little effect on the conclusions. For the standard model this means that C can be a direct product of colour, left and right permissible polynomial factors; thus

$$C = (1 + c_1 Z + c_2 Z^2 + c_3 Z^3)(1 + c_R Z_R + c_{RR} Z_R^2)(1 + c_L Z_L + c_{LL} Z_L^2) \quad (30)$$

introduces seven independent curvature coefficients c_i . The Berezinian is easily evaluated as a product of the three subgroups (trivial for SU(2)), viz.

$$\sqrt{\text{sdet } G..} = \sqrt{g..}[1 - c_1 Z + (c_1^2 - c_2)Z^2 - (c_1^3 - 2c_1 c_2 + c_3)Z^3].$$

Referring to Table 1 in reference [2], we may extract the total gauge field contributions to be

$$\frac{1}{3!2!2!} \left(\frac{l^2}{2}\right)^6 \int d^3\zeta \dots d^2\zeta_L \sqrt{-G..} \mathcal{R} = \sqrt{-g..} \left(\frac{\mathcal{A}R^{[g]}}{l^2} + \mathcal{B}\text{Tr}(F.F) + \frac{\mathcal{C}}{l^4} \right), \quad (31)$$

$$\text{where } \mathcal{A} = 4(2c_1^3 - 3c_1 c_2 + c_3)(c_{RR} - c_R^2)(c_{LL} - c_L^2), \quad (32)$$

$$\begin{aligned} \mathcal{B}\text{Tr}(F.F) &= \frac{1}{12}(3c_1^2 - 2c_2)(c_{LL} - c_L^2)(c_{RR} - c_R^2)[f^2 V^{mn} \cdot V_{mn} + 2g'^2 B^{mn} B_{mn}/3] \\ &\quad - \frac{1}{2}(2c_1^3 - 3c_1 c_2 + c_3)c_R(c_{LL} - c_L^2)g'^2 B^{mn} B_{mn} \\ &\quad - \frac{1}{4}(2c_1^3 - 3c_1 c_2 + c_3)c_L(c_{RR} - c_R^2)[g^2 W^{mn} \cdot W_{mn} + g'^2 B^{mn} B_{mn}] \end{aligned} \quad (33)$$

$$\begin{aligned} -\mathcal{C} &= 48(5c_1^4 - 10c_1^2 c_2 + 2c_2^2 + 3c_1 c_3)(c_{RR} - c_R^2)(c_{LL} - c_L^2) + \\ &\quad 8c_L(c_{RR} - c_R^2)(4c_{LL} - 3c_L^2)(2c_1^3 - 3c_1 c_2 + c_3) + \\ &\quad 8c_R(c_{LL} - c_L^2)(4c_{RR} - 3c_R^2)(2c_1^3 - 3c_1 c_2 + c_3). \end{aligned} \quad (34)$$

As perusual, $V_{mn} \equiv V_{n,m} - V_{m,n} + if[V_m, V_n]$, $W_{mn} \equiv W_{n,m} - W_{m,n} + ig[W_m, W_n]$ and $B_{mn} \equiv B_{n,m} - B_{m,n}$ retain their significance of ‘curls’ or field strengths of the three gauge fields in question.

Because C multiplies the gravitational field and we believe that gravity is parity invariant classically, we can with some confidence assume that $c_R = c_L, c_{RR} = c_{LL}$; this serves to simplify the above expressions even more and reduce the number of curvature coefficients to five. Much more importantly, we need to ensure that the normalization is identical for all the gauge fields, otherwise the stress tensor will not couple universally to all sources, as it must. Focussing on the gluon and weak boson fields, this means that $f^2(2c_2 - 3c_1^2)(c_{LL} - c_L^2) = 3g^2c_L(2c_1^3 - 3c_1c_2 + c_3)$. If we then apply the same condition for the hypercharge field, we deduce that

$$g^2 = 3g'^2 + 2g^2g'^2/3f^2,$$

which is a relation between the three coupling constants. A more meaningful version is found by remembering that $g' = g \tan \theta_w$, $e = g \sin \theta_w$, where θ_w is the weak mixing angle. Hence

$$1 = 3 \tan^2 \theta_w + 2e^2 \sec^2 \theta_w / 3f^2 \quad \text{or} \quad 4 \sin^2 \theta_w = 1 - 2e^2 / 3f^2 = 1 - 2\alpha / 3\alpha_s. \quad (35)$$

Strong chromic properties are therefore responsible for diminishing the weak angle from the purely leptonic result of $\tan^2 \theta_w = 1/3$. Since the ratio α/α_s runs, according to the renormalization group, let us estimate this reduction by choosing a weak scale of about 100 GeV when $\alpha \simeq 0.008$ and $\alpha_s \simeq 0.115$; we then obtain $\sin^2 \theta_w \simeq 0.238$. Bearing in mind that we have not considered quantum corrections, the result is not far off the standard experimental estimates^{3,4,5} hovering around 0.232.

Extracting a common factor $g^2c_L(c_{LL} - c_L^2)(2c_1^3 - 3c_1c_2 + c_3)/2$ from (32)-(34) we get

$$\begin{aligned} \int d^3\zeta \dots d^2\zeta_L \sqrt{-G} \mathcal{R} / \sqrt{-g} = & \frac{4(c_{LL} - c_L^2)}{l^2 g^2 c_L} R^{[g]} - \\ & (V^{mn} V_{mn} + W^{mn} W_{mn} + B^{mn} B_{mn}) / 4 \\ & - \frac{16}{g^2 l^4} \left[\frac{3(c_L^2 - c_{LL})(5c_1^4 - 10c_1^2 c_2 + 2c_2^2 + 3c_1 c_3)}{c_L(2c_1^3 - 3c_1 c_2 + c_3)} + (3c_L^3 - 4c_{LL}) \right] \end{aligned} \quad (36)$$

whence we may identify the Newtonian constant

$$64\pi G_N = l^2 g^2 c_L / (c_{LL} - c_L^2) \quad (37)$$

and the cosmological constant Λ via

$$\frac{\Lambda}{8\pi G_N} = \frac{16}{l^4 g^2} \left[\frac{3(c_L^2 - c_{LL})(5c_1^4 - 10c_1^2 c_2 + 2c_2^2 + 3c_1 c_3)}{c_L(2c_1^3 - 3c_1 c_2 + c_3)} + (3c_L^3 - 4c_{LL}) \right] \quad (38)$$

So far we have not succeeded in establishing a principle for restricting property curvature coefficients, and can only deduce that the c ’s are constrained by the conditions,

$$c_L \neq 0, \quad c_{LL} \neq c_L^2, \quad c_L / (c_{LL} - c_L^2) > 0$$

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and the bracketted expression in (38) is positive. The observational smallness of Λ indicates that both^d

$$4c_{LL} \simeq 3c_L^2 \text{ and } 5c_1^4 - 10c_1^2c_2 + 2c_2^2 + 3c_1c_3 \simeq 0.$$

We still have to describe the interactions of the gauge fields with the matter fields annotated in section 3. The procedure is rather automatic and hails from the frame vectors:

$$\begin{aligned} 2E_a^\zeta &= i \left(fV_{a,\lambda} - \frac{2}{3}g'B_a \right) \zeta, \quad 2E_a^{\zeta_L} = i(gW_{a,\tau} - g'B_a)\zeta_L, \quad 2E_a^{\zeta_R} = ig'B_a(\tau_3 - 1)\zeta_R \\ 2iE_a^{\bar{\zeta}} &= \bar{\zeta} \left(fV_{a,\lambda} - \frac{2}{3}g'B_a \right), \quad 2iE_a^{\bar{\zeta}_L} = \bar{\zeta}_L(gW_{a,\tau} - g'B_a), \quad 2iE_a^{\bar{\zeta}_R} = \bar{\zeta}_R g'B_a(\tau_3 - 1), \end{aligned}$$

which are the primary source of the supermetric elements (25)-(28). These frame vectors are, in this scheme, associated with the covariant derivative

$$\begin{aligned} D_a = E_a^M \partial_M &= E_a^m \partial_m + E_a^\zeta \partial_\zeta + E_a^{\bar{\zeta}} \partial_{\bar{\zeta}} + \\ &E_a^{\zeta_L} \partial_{\zeta_L} + E_a^{\bar{\zeta}_L} \partial_{\bar{\zeta}_L} + E_a^{\zeta_R} \partial_{\zeta_R} + E_a^{\bar{\zeta}_R} \partial_{\bar{\zeta}_R}. \end{aligned} \quad (39)$$

To see how this automatically produces the interactions of gauge fields with source fields, we shall simplify the presentation to start with by going to flat space, pretending there is no property curvature nor any spacetime curvature. Thus assume a Minkowskian background and a trivial curvature polynomial ($C=1$ or $c_i=0$). Consider first the four generations of leptons $\mathcal{L}, \mathcal{L}', \mathcal{L}'', \mathcal{L}'''$ where the generic doublet is $\mathcal{L}_{\{L,R\}} = (\nu_{\{L,R\}}, \ell_{\{L,R\}})$. Ignoring conjugate contributions which only serve to double the results, these leptonic doublets arise in the selfdual superfield Ψ combination as follows:

$$\begin{aligned} 2\Psi \supset & [\bar{\zeta}_L \mathcal{L}_L (1 - Z^3/6) + \bar{\zeta}_L \mathcal{L}'_L Z(1 - Z/2)/\sqrt{3}] Z_R (1 - Z_L) \\ & + [\bar{\zeta}_L \mathcal{L}''_L (1 - Z^3/6) + \bar{\zeta}_L \mathcal{L}'''_L Z(1 - Z/2)/\sqrt{3}] (1 - Z_R^2/2)(1 - Z_L), \\ & + (L \leftrightarrow R) \end{aligned} \quad (40)$$

$$\begin{aligned} 2\bar{\Psi} \supset & [\bar{\mathcal{L}}_L \zeta_L (1 - Z^3/6) + \bar{\mathcal{L}}'_L \zeta_L Z(1 - Z/2)/\sqrt{3}] Z_R (1 - Z_L) \\ & + [\bar{\mathcal{L}}''_L \zeta_L (1 - Z^3/6) + \bar{\mathcal{L}}'''_L \zeta_L Z(1 - Z/2)/\sqrt{3}] (1 - Z_R^2/2)(1 - Z_L), \\ & + (L \leftrightarrow R). \end{aligned} \quad (41)$$

In this flat limit one readily checks that

$$\int d^3\zeta \dots d^2\zeta_L \bar{\Psi} \gamma \cdot \partial \Psi = \bar{\mathcal{L}} \gamma \cdot \partial \mathcal{L} + \bar{\mathcal{L}}' \gamma \cdot \partial \mathcal{L}' + \bar{\mathcal{L}}'' \gamma \cdot \partial \mathcal{L}'' + \bar{\mathcal{L}}''' \gamma \cdot \partial \mathcal{L}'''.$$

Moreover a mass term $\bar{\Psi}\Psi$ vanishes upon integration for two reasons: chiral orthogonality and a mismatch between powers of left and right properties; this indicates

^dOne can only make guesses as to what values the curvature coefficients take. With no principle to guide us this disappointment is mitigated by the freedom afforded us by the c_i in arranging the cosmological constant to fit observation.

introduction of a Bose field which straddles chirality and cures the property mismatch too.

Upon including the frame vectors as in (39), introducing spacetime curvature and noting that the leptonic components in (40) only involve the left/right properties, we see that

$$i\gamma^a D_a \Psi \supset i\gamma^a (\partial_a + E_a \bar{\zeta}_L \partial_{\bar{\zeta}_L}) \bar{\zeta}_L \mathcal{L} + .. = \bar{\zeta}_L \gamma^a e_a^m \left[i\partial_m - \frac{1}{2}(gW_m \cdot \tau - g'B_m) \right] \mathcal{L} + .. \quad (42)$$

and similarly for the right parts. We thereby recover the standard gauge interactions for every leptonic generation.

With quarks the problem is slightly more complicated in that we must include the chromic properties; to illustrate what happens, consider the first two generations and ignore curvature (where $\mathcal{Q}\zeta\zeta$ signifies $\mathcal{Q}^1\zeta^2\zeta^3$ plus perms):

$$4\Psi \supset [(\bar{\zeta}_L^0 U_L + \zeta_L^4 D_L)Z_R + (\zeta_L^0 U'_L + \zeta_L^4 D'_L)(1 - Z_R^2/2)](1 + Z_L)\zeta\zeta(1 - Z) \quad (43) \\ + (L \leftrightarrow R) + \dots$$

This is correctly normalized in as much as

$$\int d^3\zeta \dots d^2\zeta_L \bar{\Psi} \gamma \cdot \partial \Psi = (\bar{U} \gamma \cdot \partial U + \bar{D} \gamma \cdot \partial D) + (\bar{U}' \gamma \cdot \partial U' + \bar{D}' \gamma \cdot \partial D') + \dots$$

Noting that the chromic product $\zeta\zeta$ behaves like $\bar{\zeta}$ in (43), we see that the inclusion of the frame vectors from (39) brings in

$$2D_a = e_a^m \left[2\partial_m + i(fV_m \cdot \lambda - \frac{2}{3}B_m)\zeta\partial_\zeta - i\bar{\zeta}_L(gW_m \cdot \tau - g'B_m)\partial_{\bar{\zeta}_L} - ig'B_m(\tau_3 - 1)\bar{\zeta}_R\partial_{\bar{\zeta}_R} \right] \quad (44)$$

which yields precisely the correct interactions with gluons and electroweak gauge bosons of each quark generation.

It only remains to describe what effect the property curvature polynomial C in (30) has on these standard results, where we previously set $C = 1$. For example return to the first two quark ($\mathcal{Q} = (U, D)$) generations of the superfield Ψ in (40) to discover the effect. Because E_a^M contains the factor $1/\sqrt{C}$ as well the overall factor $\sqrt{-G_{..}}$, this will induce a mixing and renormalization of the quark fields due to the various curvature coefficients. From (43) we get,

$$\Psi \propto \bar{\zeta}_L [\mathcal{Q}Z_R + \mathcal{Q}'_L(1 - Z_R^2/2)](1 + Z_L)\zeta\zeta(1 - Z) + (L \leftrightarrow R),$$

and the factorised expression

$$\frac{\sqrt{-G_{..}}}{\sqrt{C}} = \sqrt{-g_{..}} \left[1 - \frac{3}{2}c_1 Z + .. \right] \left[1 - \frac{1}{2}c_L Z_L + \frac{1}{2}(\frac{3}{4}c_L^2 - c_{LL})Z_L^2 \right] \\ \times \left[1 - \frac{1}{2}c_R Z_R + \frac{1}{2}(\frac{3}{4}c_R^2 - c_{RR})Z_R^2 \right].$$

Imposing the parity even conditions $c_R = c_L, c_{RR} = c_{LL}$, we deduce that the quark currents are the mixtures:

$$(1 + c_L/4)(1 + 3c_1/2)[\bar{\mathcal{Q}}\gamma\mathcal{Q} + x(\bar{\mathcal{Q}}'\gamma\mathcal{Q} + \bar{\mathcal{Q}}\gamma\mathcal{Q}') + y\bar{\mathcal{Q}}'\gamma\mathcal{Q}'],$$

where $x = -c_L/2$, $y = 1 + c_{LL}/2 - 3c_L^2/8$. These currents can be diagonalised by taking the combinations $\mathcal{Q} + x\mathcal{Q}'$, \mathcal{Q}' and carrying out further wave renormalizations on each of the mixed fields. It should be emphasized that these mixings and rescalings affect the kinetic energy of the quarks and their gauge interactions *equally* so no further scalings of coupling constants are needed. The very same phenomenon extends to the other quark generations as well as to leptons.

4.2. Yukawa interactions

Our next port of call is the Bose superfield Φ since it connects left to right fermion sectors. As will be seen from (21) and (22) there are numerous bosonic colour triplets and octets; but on the presumption that colour is always confined let us concentrate on the colour singlets; these contain the Higgs fields whose expectation values are added to give rise to the fermion and gauge boson masses. Consider then a hermitian superfield in flat space,

$$\begin{aligned} \Phi \supset (1 + Z^3/6)[\bar{\zeta}_L \varphi \zeta_R(1 + Z_R)(1 + Z_L) + \bar{\zeta}_L \varphi' \zeta_R(1 - Z_R)(1 - Z_L) \\ + \bar{\zeta}_R \varphi^\dagger \zeta_L(1 + Z_L)(1 + Z_R) + \bar{\zeta}_R \varphi'^\dagger \zeta_L(1 - Z_L)(1 - Z_R)] - \\ (1/\sqrt{3})Z(1 + Z/2)[\bar{\zeta}_L \varphi'' \zeta_R(1 + Z_R)(1 + Z_L) + \bar{\zeta}_L \varphi''' \zeta_R(1 - Z_R)(1 - Z_L) \\ + \bar{\zeta}_R \varphi''^\dagger \zeta_L(1 + Z_L)(1 + Z_R) + \bar{\zeta}_R \varphi'''^\dagger \zeta_L(1 - Z_L)(1 - Z_R)]. \quad (45) \end{aligned}$$

Imposing parity evenness under $L \leftrightarrow R$, so $\varphi = \varphi^\dagger$, etc. we may reduce (45) to the components

$$\begin{aligned} \Phi \supset (1 + Z^3/6)[\bar{\zeta}_L \varphi \zeta_R(1 + Z_R)(1 + Z_L) + \bar{\zeta}_L \varphi' \zeta_R(1 - Z_R)(1 - Z_L)] - \\ Z(1 + Z/2)[\bar{\zeta}_L \varphi'' \zeta_R(1 + Z_R)(1 + Z_L) + \bar{\zeta}_L \varphi''' \zeta_R(1 - Z_R)(1 - Z_L)]/\sqrt{3}, \quad (46) \end{aligned}$$

plus terms where $R \leftrightarrow L$. The normalization of the 4 possible real Higgs quartets, $\varphi = (\phi_0 I + \tau \cdot \phi)/\sqrt{2}$, then emerges correctly:

$$\int d^3\zeta \dots d^2\zeta_L \Phi^2 \propto \text{Tr}[\varphi^2 + \varphi'^2 + \varphi''^2 + \varphi'''^2] + \dots$$

Each of these four φ fields couples in the same way to gauge fields when one works out the the action $D_a \Phi D^a \Phi$ because the covariant derivative D , as in (39), acts in the expected manner on the properties ζ_L, ζ_R and the Z are blind to its action. Previously, when we were dealing with the purely leptonic case, we had just one Higgs quartet and effectively one interaction with gauge bosons. We must now confront four possible ones, but all of the same type. At this point, without a full investigation of the potential $V(\Phi)$ responsible for producing classical expectation values of $\langle \varphi \rangle$, we only know that a vacuum state arises through a linear combination of each of the four generic¹ terms $\langle \phi_0 + \phi_3 \rangle$ and we cannot say much more. In any event, when one turns on spacetime and property curvature, via $e_a{}^m(x)$ and $C(Z)$, the ensuing fields will conform to general relativity and states with the same quantum numbers will mix (and require rescaling) depending on the magnitudes of the curvature coefficients c_i . This mirrors the fermions.

The Yukawa interactions resemble those of the standard model but are yet different. Thus the standard model in its simplest form entertains a single Higgs field ϕ with independent couplings \mathbf{g}_i to each of the sources ψ_i ; in the present scheme we encounter a single Yukawa superfield^e coupling $\mathbf{g}\bar{\Psi}\Phi\Psi$ where Φ encompasses the several Higgs fields φ_i . To see how this works out in practice, study the interactions of the component fields in (46) say with the source fields for leptons and quarks as in (40 and (43). The calculations are a bit messy but straightforward: one simply has to collect the correct powers of property before integrating over them. In the quark sector, with reference to (43) and (46), we obtain such interactions as

$$\begin{aligned} \int d^3\zeta \dots d^2\zeta_L \Psi\Phi\Psi &\propto (\bar{Q}_R + 2\bar{Q}'_R)(\varphi - \varphi''/\sqrt{3})(Q_L + 2Q'_L) \\ &\quad + \bar{Q}_R(\varphi' - \varphi'''/\sqrt{3})Q_L + (L \leftrightarrow R). \end{aligned} \quad (47)$$

For leptons we get a similar result:

$$\begin{aligned} \int d^3\zeta \dots d^2\zeta_L \Psi\Phi\Psi &\propto \bar{\mathcal{L}}\varphi\mathcal{L} + (\bar{\mathcal{L}} + 2\bar{\mathcal{L}}'')\varphi'(\mathcal{L} + 2\mathcal{L}'') \\ &\quad + 2\bar{\mathcal{L}}'\varphi\mathcal{L}' + 2(\bar{\mathcal{L}}' + 2\bar{\mathcal{L}}''')\varphi'(\mathcal{L}' + 2\mathcal{L}''') \\ &\quad + 2[\bar{\mathcal{L}}'\varphi''\mathcal{L}' + (\bar{\mathcal{L}}' + 2\bar{\mathcal{L}}''')\varphi'''(\mathcal{L}' + 2\mathcal{L}''')] / \sqrt{3}. \end{aligned} \quad (48)$$

Introducing curvature of spacetime through $\det e$ renders the answers generally covariant, while inclusion of property curvature $C(Z)$ causes the various leptonic and quark components to mix even more through the curvature coefficients c_i , as we saw for fermions, but *without disturbing the gauge couplings*. The whole edifice becomes rather complicated and warrants future computational analysis though it is conceptually simple.

5. Concluding Remarks

This work represents the culmination of a series of articles exploring the picture that the “extra dimensions” which everyone is seeking to append to spacetime are simply the properties of a system, represented by scalar anticommuting complex variables. In earlier articles we investigated what happens when we added the properties successively and we gradually gained an understanding of how the scheme operates; with this knowledge we have in this paper incorporated the full gamut characterising the SM: chromicity, left and right electricity/neutrinity. In this scenario an event—where something necessarily changes—is fully described by where, when, and *what* happens.

A consequence of this approach is that gravity and the gauge interactions of the standard model are fully unified in a generalised metric containing curvature of spacetime as well as property. The ensuing field theory and particle spectrum

^eBelow we have ignored the duality choices mentioned at the start of section 3.2 since these do not couple to the fermion superfield Ψ upon integrating over ζ_L and ζ_R , at least in the flat limit.

demands the imposition of Grassmannian duality, as explained in sections 2 and 3. Even if our choices of dualities within the subgroups of the full group turn out to be awry, the need for some kind of duality is imperative and emergence of generations is inevitable. However, assuming our choices are the correct ones, we were able to accommodate the known particle generations, but in the process we were led to predict other generations of quarks and leptons, and especially the existence of a quark carrying charge $-4/3$, accompanying t & b say in a weak isospin triplet. Another feature was the emergence of totally sterile scalar states (8 of them), singlets of all the gauge groups and therefore decoupled from standard matter; these may or may not have some connection with dark matter—it is too early to say. The marriage of gravity to the gauge fields via the frame vectors and insistence on the universal character of coupling to gravity resulted in the prediction of a weak mixing angle which is close to experiment.

Apart from researching the mechanism generating masses through expectation values of at least four standard Higgs fields which couple to the sources, the property curvature coefficients stick out as the only vague feature of this scheme. We have been unable to formulate a principle which determines the c_i , which in turn fix the cosmological constant, etc. What we do know is that pure property transformations like $\zeta \rightarrow \zeta f(\bar{\zeta}\zeta)$, which do not affect gauge variations, produce nasty looking off-diagonal elements in the property-property sector via the altered metric $d\bar{\zeta}\zeta d\bar{\zeta}$, etc. Conversely those property transformations can be used to eliminate such unpleasant contributions, but will not fix the curvature coefficients. This is the most pressing problem confronting us. Only after its proper resolution might we turn to the BRST quantization since there seems to be a natural place for it within the current framework. A separate issue is the calculation of masses and mixings, where c_i enter once again. In our opinion, It seems unlikely that we will be able to cover the huge and diverse range of masses (from neutrinos to the top quark and mixings) with this set of c_i and various $\langle\varphi\rangle$; most probably some dynamical mechanism will be needed to produce the masses of the lighter leptons through their weak interactions and the c_i may themselves be expectation values of additional scalar fields.

If right gauge bosons are experimentally discovered, they are readily incorporated by introducing additional frame vectors affecting the right properties ζ_R . In the end like everyone else we are hostage to experiment and to what the LHC and future colliders will reveal. It is entirely possible that this whole scheme will fail miserably by not agreeing with observations and thus turn out to be a figment of imagination. Hopefully some of its ingredients may survive but only time will tell.

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Brian Kenny has been urging me over the years to take the ‘next step’ in this approach to unification. I thank him for his keen interest and continued faith in this program.

Appendix A. Rule for N properties integration

In the text we are confronted with integration over properties $\zeta, \bar{\zeta}$. Our convention for handling this is as follows. The Grassmannian measure is to be taken in the order $(d\zeta^1 d\bar{\zeta}^1)(d\zeta^2 d\bar{\zeta}^2) \dots (d\zeta^N d\bar{\zeta}^N) = (-1)^{<N/2>} (d\zeta^1 d\zeta^2 \dots d\zeta^N)(d\bar{\zeta}^1 d\bar{\zeta}^2 \dots d\bar{\zeta}^N)$, where $<N/2>$ signifies the integer part of $N/2$. We define invariants $Z \equiv \zeta^{\bar{\mu}} \zeta^{\mu}$ for each of the subgroups (with appropriate labelling on Z) such that for $SU(N)$,

$$\int (d\zeta^1 d\bar{\zeta}^1)(d\zeta^2 d\bar{\zeta}^2) \dots (d\zeta^N d\bar{\zeta}^N) Z^N = N!$$

Appendix B. Rule for $SU(N)$ duality

If one substitutes $(r, s) \rightarrow (N - s, N - r)$ in the expansion of a superfield in powers $(\zeta)^r (\bar{\zeta})^s$ it is a fact that the associated $SU(N)$ representations are duplicated, corresponding to reflection about the cross-diagonal. We call the act of cross-diagonal reflection the ‘duality’ operation and indicate it by the symbol \times . Specifically our rule for determining the factors arising from this reflection is given by

$$(\zeta^{\bar{\mu}_1} \dots \zeta^{\bar{\mu}_r} \zeta^{\nu_1} \dots \zeta^{\nu_s})^\times = (\epsilon^{\bar{\mu}_1 \dots \bar{\mu}_r \rho_1 \dots \rho_{N-r}} \zeta^{\rho_{N-r}} \dots \zeta^{\rho_1}) (\zeta^{\bar{\sigma}_{N-s}} \dots \zeta^{\bar{\sigma}_1} \epsilon^{\sigma_1 \dots \sigma_{N-s} \nu_1 \dots \nu_s}) / (N-r)!(N-s)!$$

This looks rather complicated, but three examples may make this rule clearer. For instance consider colour ($N = 3$); then the dual of a typical octet element is associated with a - sign as follows:

$$(\zeta^{\bar{1}} \zeta^2)^\times = \epsilon^{\bar{1}\bar{2}\bar{3}} \zeta^3 \zeta^2 \zeta^{\bar{1}} \zeta^{\bar{3}} \epsilon^{312} = -\zeta^{\bar{1}} \zeta^2 \zeta^{\bar{3}} \zeta^3 = -\zeta^{\bar{1}} \zeta^2 Z,$$

whereas for the singlet product of the three colours there is no sign change:

$$(\zeta^{\bar{1}} \zeta^{\bar{2}} \zeta^{\bar{3}})^\times = \epsilon^{\bar{1}\bar{2}\bar{3}} \zeta^3 \zeta^2 \zeta^{\bar{1}} \epsilon^{123} = \zeta^{\bar{1}} \zeta^{\bar{2}} \zeta^{\bar{3}}.$$

Similarly for the triplet,

$$(\zeta^{\bar{1}} Z)^\times = (\zeta^{\bar{1}} (\zeta^{\bar{2}} \zeta^2 + \zeta^{\bar{3}} \zeta^3))^\times = \epsilon^{\bar{1}\bar{2}\bar{3}} \zeta^3 \zeta^{\bar{3}} \zeta^{\bar{1}} \epsilon^{132} + \epsilon^{\bar{1}\bar{3}\bar{2}} \zeta^2 \zeta^{\bar{1}} \zeta^{\bar{2}} \epsilon^{213} = -\zeta^{\bar{1}} Z.$$

By these sorts of manipulations we can ascertain the duality signs for all the entries in the expansion table, as stated in the text; applying selfduality or anti-selfduality can the sometimes serve to eliminate some of the representations sitting on the cross-diagonal. Finally, it goes without saying that dualizing twice results in the identity operation.

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